Superellipse Fitting for the Recovery and Classification of Mine-Like Shapes in Sidescan Sonar Images

Esther Dura, Judith Bell, and Dave Lane

Abstract—Mine-like object classification from sidescan sonar images is of great interest for mine counter measure (MCM) operations. Because the shadow cast by an object is often the most distinct feature of a sidescan image, a standard procedure is to perform classification based on features extracted from the shadow. The classification can then be performed by extracting features from the shadow and comparing this to training data to determine the object. In this paper, a superellipse fitting approach to classifying mine-like objects in sidescan sonar images is presented. Superellipses provide a compact and efficient way of representing different mine-like shapes. Through variation of a simple parameter of the superellipse function different shapes such as ellipses, rhomboids, and rectangles can be easily generated. This paper proposes a classification of the shape based directly on a parameter of the superellipse known as the squareness parameter. The first step in this procedure extracts the contour of the shadow given by an unsupervised Markovian segmentation algorithm. Afterwards, a superellipse is fitted by minimizing the Euclidean distance between points on the shadow contour and the superellipse. As the term being minimized is nonlinear, a closed-form solution is not available. Hence, the parameters of the superellipse are estimated by the Nelder–Mead simplex technique. The method was then applied to sidescan data to assess its ability to recover and classify objects. This resulted in a recovery rate of 70% (34 of the 48 mine-like objects) and a classification rate of better than 80% (39 of the 48 mine-like objects).

Index Terms—Classification, mine-like objects, recovery, sidescan, sonar, superellipse fitting.

I. INTRODUCTION

Within the last decade autonomous underwater vehicles (AUVs) have become a very attractive technology for use in scientific, industrial, and military applications. The spectrum of applications in which AUVs are involved is considerable. These include surveying the pipes and cables of telecommunications and oil companies, seafloor mapping, mine counter measure (MCM) operations and sea recovery operations. Given the potential applications, a self contained, intelligent, decision making AUV is the current goal in underwater robotics. In particular, correct visualization and automatic interpretation of sonar images can provide AUVs with a wealth of information, facilitating planning, and decision making, and therefore, enhancing vehicle autonomy.

In recent years, countermine warfare has become an increasingly important issue and the inherently covert nature of AUVs make them an appealing platform for shallow-water MCM operations. Images produced from sidescan sonars mounted on the vehicles are generally used for these purposes. As a result of the low levels of contrast apparent in the images, the shadow cast by objects within these images frequently appears more prominent and gives better clues about the shape and the size of the object than the highlight. For this reason, much of the research applying image processing and pattern recognition techniques to MCM concentrates on analyzing the shadow information as it is crucial for computer-aided detection (CAD) and computer-aided classification (CAC) operations. Although much research has been carried out on CAC [1]–[3], the problem of classifying mine type and orientation (CAC operations) has not been widely addressed.

The main contribution of this work lies in the use of a superellipse fitting procedure for CAC operations to recover and classify mine-like objects in sonar imagery. Superellipses provide a compact and efficient way of representing the shadows cast by different mine-like shapes. By simply varying the squareness of the superellipse function, different shapes such as ellipses, rhomboids, and rectangles can be easily generated. It is an appealing alternative to feature-based and image-based approaches because it provides a smart and efficient solution. Although up to date superellipse models have been previously applied for the detection of primitives in mechanical objects in video images and segmenting curves into several superellipses [4], they have not been previously employed in the context of mine-like object recovery and classification.

In this work, superellipse detection is performed with the dual aims of recovering and classifying mine-like shadow shapes.

• Shape recovery: The shadow produced in sonar images may be not well defined due to speckle noise and the environment (spurious shadows, sidelobe effects, and multipath returns). This may result in missing and noisy boundaries, which makes identification difficult. When fitting a superellipse to the data, the final configuration aids in revealing the closest shape and location of the object. The recovery process is particularly important to aid 3-D reconstruction of mine-like objects [5].
• **Shape classification:** This determines which of the class representatives is most similar. In this work, a parameter of the superellipse, known as the squareness parameter, determines a variety of shapes including rhomboid, rectangular, and spherical shapes, and is exploited here to discriminate between them. In addition, other superellipse parameters, for example, the axis lengths and the degree of rotation, may be used to characterize the dimensions and orientation of the mine.

It is assumed that this classification procedure is part of a broader detection and classification system and that the region of interest has been obtained on the basis of some prior detection method (such as that proposed by Reed *et al.* [6]) and that man-made objects have been separated from non-man-made objects.

### II. RELATED WORK

In general, classical models consisting of feature extraction and subsequent classification have been widely used in CAC operations. First, using a presegmented shadow, a set of features is extracted from the shadow. This allows the representation of the image by a few descriptors providing relevant information about the extracted shadow rather than the whole set of pixels. Then, the set of features is normally presented as input to a classifier. Commonly used feature vectors include Fourier descriptors [7], [8], shape descriptors [9]–[14], and intensity descriptors [15], [11], [8], [13]. Recently, Zerr *et al.* [16] and Myers [17] used the object height profile of shadow information as a feature vector. Although feature-based methods are appealing, the performance of the classifiers depends, to a significant extent, upon the ability of the chosen features to accurately represent all of the characteristics of the shadows.

Alternative model-based approaches have been proposed for the classification of mine-like objects. Based on the observation that mine-like objects cast a regular and easily identifiable geometrical shape, *a priori* knowledge can be taken into account to detect instances of shapes of a determined object. In particular, the cast shadow of a sphere is an ellipse and the cast shadow of a cylinder is a parallelogram in most of the cases. Therefore, different templates can be defined to detect ellipses and parallelograms. This approach can be used for classifying and recovering cast shadows more efficiently than feature-based approaches if *a priori* information can be introduced.

A model-based approach that combines available properties of the shape (as a prior model) and an observation model (as a likelihood model) was proposed by Mignotte *et al.* [18], to detect and classify mines in sonar imagery. In such terms, they defined a prototype template, along with a set of admissible linear transformations, to take into account the shape variability for every type of mine. They also defined a joint probability density function (pdf), which expresses the dependence between the observed image and the deformed template. The detection of an object class was based on an objective function measuring how well a given instance of deformable template fits the content of the segmented image. The function was minimal when the deformed template exactly coincides with the edges of the scene’s Fourier coefficients. In particular, this implementation performed very well when objects were occluded in the scene.

Alternative approaches have been proposed. Fawcett [20] worked directly with the image itself as a feature. The basic approach consisted of applying a principal component analysis to the image and then a discriminative analysis was used to determine the vectors that best discriminate the object class. Afterwards these vectors were used to cluster the images. Quidu *et al.* [21] also used a similar technique relying on the junction of the segmentation and classification steps by using Fourier descriptors and genetic algorithms. Although the results presented in these works are promising, only synthetic data were used.

In this work, a model-fitting approach is also advocated for the recovery and classification of the shadow information of mine-like objects. The scope of the work presented by Balasubramanian and Stevenson [8] is extended by modeling the mine-like shadow with a superellipse.

In sonar imagery, mine-like objects, due to their regular shape, tend to produce a shadow that also represents a regular geometrical shape such as those illustrated in Fig. 1. In particular, the shadow cast by a spherical mine almost always is an ellipse. For cylindrical mines, the associated shadow may be a rhomboid, an ellipse, or a rectangle. Therefore, two different types of templates, as stated by Mignotte *et al.* [18], could be defined to characterize these shapes. The main drawback of this approach [18] is that it may be computationally expensive because different templates must be defined to describe different shapes. Consequently, to determine the presence of a mine-like object, all of the defined templates have to be searched.

The superellipse provides a more compact and interesting approach for representing this variety of shapes. With a simple analytical function composed of small number of parameters (as described in Section III-B), a wide range of objects including ellipses, rectangles, rhomboids, ovals, and pinched diamonds can be represented. In addition, the range of shapes described by a superelliptical model can be extended by adding parameters to describe model deformation [5].

### III. SUPERELLIPSE MODEL FITTING

#### A. Superellipses

Superellipses are a special case of curves that are known in analytical geometry as Lamé curves [22], named after the math-
ematician Gabriel Lamé who described these curves in the early 19th century. Piet Hein popularized these curves for design purposes in the 1960s and named them superellipses. Barr [23] generalized the superellipses to a family of 3-D shapes named superquadrics, which became very popular in computer graphics, and in particular, in computer vision, with the work of Pentland [24]. They can represent many closed 2-D and 3-D shapes in a straightforward and natural way using only a few parameters, and moreover, simple deformations can be applied to extend their modeling capabilities.

B. Definition of Superellipses

A superellipse centered on the origin, with its axes aligned with the coordinate system, can be represented by the following implicit equation:

\[
\left(\frac{x}{a}\right)^{\frac{2}{e}} + \left(\frac{y}{b}\right)^{\frac{2}{e}} = 1
\]  

where the lengths of the axes are given by \(a\) and \(b\) and the squareness is determined by \(e\). Equation (1) involves a complex root for negative values of \(x\) and \(y\). Due to the symmetry of the superellipse, this can be avoided using the absolute values of \(x\) and \(y\) as

\[
\left(\frac{|x|}{a}\right)^{\frac{2}{e}} + \left(\frac{|y|}{b}\right)^{\frac{2}{e}} = 1.
\]  

This would produce the curve in the positive \(x\) and \(y\) quadrant. The curve can then be reflected into the other quadrants.

Fig. 2 shows a superellipse with \(a = 2b\) and \(e\) equal to 0.1, 0.5, 1.0, 2.0, and 5.0. A value of \(e = 0.1\) produces a rectangle with round corners (very low values of \(e\) result in a perfect rectangle), \(e = 1\) produces an ellipse, \(e = 2\) produces a rhomboid, and for values larger than 2, it produces pinched diamonds (very large values result in a cross).

The function

\[
F(x, y, a, b, e) = \left(\frac{x}{a}\right)^{\frac{2}{e}} + \left(\frac{y}{b}\right)^{\frac{2}{e}}
\]  

is called the “inside–outside” function because its value determines whether a given point \((x, y)\) lies inside, right on the boundary, or outside the superellipse contour

\[
F(x, y, a, b, e) = \begin{cases}  
< 1: & \text{inside} \\
= 1: & \text{on the contour} \\
> 1: & \text{outside}
\end{cases}
\]  

This can also be defined in parametric form by

\[
x(t) = a \text{sgn}(\sin t) \sin \frac{\pi t}{e} \\
y(t) = b \text{sgn}(\cos t) \cos \frac{\pi t}{e}
\]

where \(0 \leq t \leq 2\pi\).

To have real values that can be plotted for every meaningful value of \(t\), (5) and (6) are implemented as

\[
x(t) = a \text{sgn}(\sin t) |\sin (\pi t/e)| \\
y(t) = b \text{sgn}(\cos t) |\cos (\pi t/e)|
\]  

where \(\text{sgn}\) represents the sign.

C. Related Work on Superellipse Fitting

Superellipse curves extend the scope of conic sections such as ellipses, circles, and lines. Two different approaches have been explored for fitting superellipses to data points: point distribution models (PDMs) and nonlinear least square minimization on an appropriate error of fit function.

Fitting superellipses by PDM was investigated by Pilu [25]. PDM is a term coined by Cootes et al. [26] to indicate statistical finite-element models built from a training set of labeled contour landmarks of a large number of shape examples. The key idea of the work proposed in [25] is to use a mathematical model, which itself represents a class of shapes, to train a PDM. The training set is built from randomly deformable superellipses and then a method is used for fitting these models to data points. This approach represents a good balance between ease of fitting and representational power. However, the computational requirements are high as a large training data set needs to be generated.

Few authors have used the superellipse for curve representation, however segmentation of range images into patches by using various nonlinear least square minimization techniques on
an error of fit function [27]–[29] has been well studied. Most researchers used the inside–outside algebraic function and some variations of it as an error of fit. However, as pointed out in [27], this error function introduces a high-curvature bias that often leads to counterintuitive results. Based on these results, an error of fit function relying on the Euclidean distance was suggested by Gross [27], providing better results [30]. These results inspired the work presented by Rosin and West [4].

Rosin and West [4] started working on the problem of superellipse fitting by using a Powell’s optimization technique to minimize an appropriate error metric based on the Euclidean distance. However, the technique presented in this earliest work was computationally expensive as it was a 6-D optimization problem. Later, Rosin [31] revised this problem setting all the parameters using either an ellipse or a rectangle except for the squareness, which was found by 1-D optimization. Also, nine error of fit measures were compared. Testing on synthetic data and real data contaminated with noise revealed that the 1-D optimization was faster, however when substantial amounts of noise and occlusion were added, the 6-D optimization technique [4] performed much better than 1-D optimization techniques. Hu [32] used a similar technique to the one proposed by Rosin and West [4]. The main difference lay in the initialization of the parameters. Whereas [4] estimated the orientation and the translation by principal moments and the main axis by fitting an ellipse, Hu [32] estimated them by computing the zeroth harmonic of Fourier descriptors. The similarity of the results showed that any of the techniques can be applied. Thus, both techniques are good methods for superellipse detection.

D. Technique Implemented for Fitting Superellipses

In this work, superellipses are fitted by finding the set of parameters that minimize the error measure proposed in [4] and [27]. In essence, the method is equivalent to the one proposed in [4], however both the aim and the optimization technique are different.

Metrics similar to those used for the ellipse fitting [33] and polynomial fitting [34] have been investigated for fitting a superellipse to a contour of points. The simplest measure is the algebraic distance given by

\[ Q(x, y) = \left( \frac{x}{a} \right)^\frac{\alpha}{2} + \left( \frac{y}{b} \right)^\frac{\beta}{2} - 1. \]  

(9)

However, experimental results showed that a high curvature bias is involved, in which the algebraic distance from a point to the superellipse is underestimated. Other methods such as weighting the algebraic distance [35] by its gradient have been proposed to cope with this problem, but they also proved to be unstable.

Instead, as proposed in [4] and [27], it was chosen to minimize the Euclidean distance \( d_i \) from a data point \((x_p, y_p)\) on the shadow contour to the point \((x_s, y_s)\) on the superellipse along the line that passes through \((x_p, y_p)\) and the center of the superellipse \((0, 0)\) (see Fig. 3), where

\[ d_i = \sqrt{(x_p - x_s)^2 + (y_p - y_s)^2}. \]  

(10)

Equations (10)–(12) involve evaluating complex roots for negative values of \( x \) and \( y \). Nevertheless, this can again be avoided by using absolute values of \( x \) and \( y \). This produces a solution that is evaluated in the positive \( x \) and \( y \) quadrant. The solution can then be reflected into the other quadrants by determining the quadrant in which the point lies.

The previous equation has been evaluated with the contour centered on the origin. Nevertheless, to allow for rotation \( (\theta) \) and translation of the center of the superellipse to the point \((x_c, y_c)\). (9) should be modified to

\[ Q(x, y) = \left( \frac{x - x_c}{a} \right)^\frac{\alpha}{2} \cos \theta + \left( \frac{y - y_c}{b} \right)^\frac{\beta}{2} \sin \theta - 1. \]  

(12)

which would result in the modification of (2) and (12), and therefore, in more complex calculations. Instead it was decided to keep the superellipse centered at the origin and aligned with the coordinates axes. Hence, when fitting data, rather than transforming the superellipse, the data is inversely transformed to fit the model.

Because the term being minimized is nonlinear, a closed-form solution is not available. The Nelder–Mead simplex technique [36], which requires simply the term being minimized, is therefore used. The advantage of using this is that it only requires function evaluations, not derivatives. Also compared to other optimization techniques, this algorithm is a simple (numerically less complicated), robust, and well-tried method for underconstrained nonlinear optimization.

With such iterative techniques, it is important to provide a good initial estimate of the superellipse parameters. In this work, the initial values of the axis lengths \((a, b)\), the rotation \( (\theta) \), and the translation parameters \((T_x, T_y)\) are found by fitting an ellipse to the data using the method proposed in [37].
This provided a reasonable initial estimate of all the parameters except for the squareness. The squareness was initialized to 1.9, which corresponds to a parallelogram shape. Preliminary experiments showed that initializing the squareness to this value avoided converging in a local minimum. These initial values were then used as input to the Nelder–Mead technique described above, where each parameter $\alpha, \beta, \theta, T_x, T_y, \varepsilon$ was reestimated iteratively until the cost function was minimized providing a superellipse with the best fit to the data.

The algorithm relies on the fact that the data set does not contain outliers. This is particularly important to consider as the outlying data gives an effect so strong in the minimization that the estimated parameters may be distorted. As the resulting contour points are corrupted in most of the cases by some artifacts such as spurious shadows and the speckle noise effect, consequently resulting in outliers, a class of robust M-estimators was considered. The M-estimators attempt to reduce the effect of outliers by replacing the cost function by another version of the original cost function.

For this particular case, the cost function $d_i$ is replaced by another cost function $\hat{d}_i$:

$$d_i = \sum_{i=1}^{N} \rho(d_i)$$

(14)

where $\rho$ is a symmetric, positive–definite function with a unique minimum at zero and $N$ is the number of points. Several distributions have been proposed, because selecting a distribution is difficult and in general rather arbitrary. Reasonably good results were obtained by adopting the Geman–McClure distribution

$$\rho(z) = \frac{z^2}{1 + z^2}.$$  

(15)

Fig. 4 illustrates the stages of the evolution of the superellipse contour during the Nelder–Mead simplex procedure on a real sidescan image of a rectangular shape on the seabed. The value of $\varepsilon$ at each of these stages is also shown.

E. Extraction of the Contour

Before the superellipse detection procedure, several steps are required to extract the data points, as illustrated in Fig. 5. First, the image is segmented by an unsupervised Markov random field (MRF) algorithm [38]. Second, the image is labeled to search for the largest region, which corresponds to the mine shadow. Afterwards an opening morphological operator with a $3 \times 3$ structural element is applied to the region to remove spurious shadows that perturbed the object shape. Finally, the contour, which contains the data points to be fed into the superellipse detection algorithm, is extracted by using a simple boundary following algorithm [39]. At this point, it is assumed that a set of image points plausibly belonging to a superellipse has been found.

A contour extraction approach was employed as this provided a robust and fast solution for real-time applications. To extract the contour points, an accurate and reliable segmentation or edge map is required. The extraction of edges using edges operators from sonar images is difficult due to the presence of speckle noise. However, it has been shown that MRF algorithms are robust and well suited for the segmentation of sonar images into shadow and reverberation [38], [40], [41]. Hence, an MRF segmentation algorithm was used to extract the shadow and from it the contour points [38].
IV. EXPERIMENTAL RESULTS

A. Data Set

In the following, the performance of the method is qualitatively demonstrated with real data. The data was collected in May 2001 by Groupe d’Etudes Sous-Marines de L’Atlantique (GESMA) near Brest, France, with the Klein 5400B multibeam sidescan sonar. The sidescan sonar operated at a frequency of 455 kHz. The system was towed at an average speed of 5 kn by the GESMA research vessel Aventuriere II.

Forty eight region of interest images of targets were extracted from the data. These images corresponded to 27 images with rectangular- and rhomboid-like shadow shapes cast by a cylindrical object and 21 images with spherical-like shadow shapes cast by a sphere lying on a pebble seabed. The cylindrical object was 2 m long with a radius of 0.5 m and the spherical object had a radius of 1 m. The images of the targets were acquired at different azimuth angles and ranges. The dimensions of the extracted images were 256 x 128 pixels and had a resolution of 3.3 cm in both x (across range) and y (along range).

B. Recovery

The superellipse fitting procedure described in Section III was applied to all 48 images. Figs. 6 and 7 display some of these results. For each case, the left-hand side column presents the original image, the MRF segmentation is shown in the center column, and the resulting fitted superellipse is in the right-hand side column with the corresponding squareness value. It can be seen that in the majority of the examples the outline of the shadow is accurately recovered.

However, in some of the cases, in spite of not accurately representing the dimensions of the recovered shape, they converged to the right shape. This is particularly important for classification purposes as will be discussed in the next section. It is also worth noting how well this technique recovered incomplete shapes with a very irregular contour, as illustrated in Figs. 6(c.1) and 7(c.2).

The recovery rate, which is the percentage of images where the superellipse fits well to the boundaries of the shadows, resulted in 70.8% (34 of 48 images were well recovered). The degree of fit was made qualitatively through visual inspection by looking how well the superellipse was fitting to the boundaries of the shadow. The recovery rate may be different from the classification rate, because the correct shape may be identified even when the superellipse does not have the correct physical dimensions (i.e., it is not recovered well).

C. Classification

One of the primary aims of fitting a superellipse to data contour points is to aid the classification task. In this section, the squareness parameter, which determines the shape of the superellipse, is exploited, sidestepping the use of feature extraction and classification procedures commonly used in the MCM operations, for classification purposes. Based on the observation that by varying the squareness at certain ranges specific shapes are generated (see Fig. 8), the classification procedure relies on the following decision rule.

- If $\varepsilon < 0.6$ or $1.2 \leq \varepsilon < 2.5$, the cast shadow is a parallelogram.
- If $0.6 \leq \varepsilon < 1.2$, the cast shadow is spherical.

In particular, for the case of parallelogram shapes, $\varepsilon$ may also aid in identifying the direction of travel of the sonar with respect to the mine-like object by looking at the skewness of the shape. If $\varepsilon$ varies between 0 and 0.6, this signifies that there is no skewness of the shape (square- or rectangle-like shapes), and therefore, the direction of ensonification is orthogonal to the object’s main axis. On the other hand, when $\varepsilon$ varies between 1.2 and 2.5, there is skewness (rhomboid shapes), and therefore, the direction of ensonification is not orthogonal to the main axis of the object.

To test the performance of the classification rule, it was applied to the previously described data set. All images were preprocessed using the algorithm presented in the previous section. The initial value of $\varepsilon$ was set to 1.9.

Table I shows the theoretical and estimated $\varepsilon$ for the cylindrical and spherical objects. According to the classification rule seen above (see Fig. 8), the convergence to a determined value relates the shadow to classifying the shadow as rectangular, rhomboid, or spherical. It can be observed that in particular for
the case of the cylinder for $0^\circ$, $178^\circ$, and $185^\circ$ angles of view, the convergence value is less than 0.6 and hence classifies the shapes as rectangular. Although the correct shape should be a perfect rectangular shape, the estimated values represent a rectangular shape with curved corners. For the $89^\circ$ angle of view, the superellipse did not converge to the right value; it converged to a square shape ($\varepsilon = 0.0157$), whereas the right shape should be an ellipse with $0.6 \leq \varepsilon < 1.2$. For the rest of the cases, the shape presented some skewness, and therefore, $\varepsilon$ varied between 1.2 and 2.1. For the case of the sphere, it can be observed that the majority of the $\varepsilon$ values were well estimated lying within the range 0.6–1.2. It is worth highlighting that depending on the slant range to the sphere, the shadow length and therefore the shape generated varied. This affected the $\varepsilon$ convergence value as can be seen in Fig. 7(c.2) for an angle of $67^\circ$, where the elliptical shadow shape is not so well defined resulting in a value of $\varepsilon$ of 0.85. However, in the criteria used for the classification, this shape is still considered an elliptical shape. On the other hand, for well-defined elliptical shadow shapes, such as the one depicted in Fig. 7(c.4), the algorithm converged with $\varepsilon = 0.95$, which is very close to a perfect elliptical shape.

Fig. 9 illustrates the classification results. In summary, 80.95% (17 of the 21 mine-like objects) of the images containing spherical-like shapes and 81.4% (22 of the 27 mine-like
objects) of those containing rectangular- and rhomboid-like shapes were classified correctly. This resulted in a total of 81.25% (39 of the 48 mine-like objects) over all the images.

When discriminating between rhomboid and rectangular shapes, the total percentage of correct classification dropped to 75% (36 of the 48 mine-like objects). All 3 of the square shapes (100%) and 16 of the 24 rhomboid shapes (66.6%) were classified correctly, resulting in a total classification rate of 70.37% for the images containing rhomboid- and rectangular-like shapes.

Of the 33.3% rhomboid-like images misclassified, 62% of these converged to a rectangular-like shape and 37% to spherical-like shapes. In this case, it is preferable for the algorithm to misclassify the shapes as rectangular because this is closer than the sphere shape to the correct classification of rhomboid.

The classification could then be improved using the additional superellipse parameters. These would provide an indication of the size of the object from the axis lengths, and its orientation on the seabed. This would require a calibration of the technique using the resolution of the sonar, and the range of the target from the sensor. This could assist in the elimination of some false alarms, if their other superellipse parameters were found to be unrealistic of typical mine dimensions. Further work could also look at subsequently fitting a superellipse to the highlight as well as the shadow to extract further information about the target.
The classification procedure may not be able use the squareness parameter, $\varepsilon$ in isolation, without considering the degree of fit obtained from the fitting procedure or a prior detection stage to eliminate nonmine-like targets. The examples shown in Fig. 10 show two objects that are not mines. In the first case, the superellipse converged to a parallelogram ($\varepsilon = 1.683$), however the degree of fit obtained from the cost measure would have rejected the object as mine-like. In the second case, the object would be classified as spherical ($\varepsilon = 0.918$), and would require further analysis to classify correctly as not mine-like because visually it appears to display mine-like characteristics.

V. CONCLUSION AND FURTHER RESEARCH

This work presented a simple approach for the classification and recovery of man-made object shapes in sidescan sonar images using a superellipse template matching scheme. The approach used a priori knowledge of the geometry of the cast shadow in sidescan sonar images. The method was tested on a large number of noisy sidescan images providing an overall recovery rate of 70% (34 of the 48 mine-like objects) and a classification rate of 81% (39 of the 48 mine-like objects). The results indicate that this may be a feasible approach for object classification purposes for use in combination with a man-made object detection system. The technique is applicable to high-resolution sidescan images, where the shadow region is represented by several pixels in either direction. The resolution of the sonar will determine the smoothness of the shadow contour, which
TABLE I

Theoretical and Experimental Values for $\varepsilon$ for the Cylinder and the Sphere Seen Under Different Angles of Views. For the Cylinder, the Results Are Classified in Three Categories: 1) Within the Expected Range, 2) Cylinder But in Another Range Window, and 3) Outside the Expected Shape Range. Those in Category 2) Are Annotated by an Asterisk. The Ones in Category 3) Are Indicated in Bold. For the Sphere, the Results Are Classified in Two Categories: 1) Within the Expected Range and 2) Outside the Expected Shape Range (Annotated by an Asterisk).

<table>
<thead>
<tr>
<th>Angle of View</th>
<th>Cylinder theoretical $\varepsilon$</th>
<th>Cylinder estimated $\varepsilon$</th>
<th>Angle of View</th>
<th>Sphere theoretical $\varepsilon$</th>
<th>Sphere estimated $\varepsilon$</th>
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<td>0°</td>
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<td>0°</td>
<td>$0.6 \leq \varepsilon &lt; 1.2$</td>
<td>1</td>
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<td>115°</td>
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can be extracted and will have implications on the degree of fit to the superellipse. However, the technique provides an alternative to traditional feature-based or image-based approaches that require a suitable training set.

Although the work presented in this paper has concentrated on specific mine-like shapes, with further extensions, the potential of the superellipse could be expanded. In particular, the superellipse could also represent the shadows cast by truncated cone and pipeline shapes with the inclusion of bending and tapering transformations [5].

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REFERENCES


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